Standard Error of Sample proportion :-

Let P be the proportion of sample given by

$$p = \frac{x}{n}$$

Where x is no. of success in n trial
Hence if n is sample size the variance of the sample proportion is

$$V(p) = V\left(\frac{x}{n}\right) \qquad \left[\text{as } V(cx) = c^2 V(x)\right]$$

$$= \frac{1}{n^2} V(x)$$

as we know $V(x) = \sigma^2$

$$\therefore V(p) = \frac{\sigma^2}{n^2} \quad \text{--- (1)}$$

But $\sigma^2 = E[x - E(x)]^2 = E(x^2) - [E(x)]^2$ --- (2)
Now we solve $E(x^2)$ & $(E(x))^2$

$$E(x^2) = E[x(x-1) + x]$$

$$= \sum_{x=0}^{n} [x(x-1) + x] \, p(X=x)$$

$$= \sum_{x=0}^{n} x(x-1)\, p(X=x) + \sum_{x=0}^{n} x \, p(X=x)$$

& $P(X=x) = \binom{n}{x} p^x q^{n-x}$ is p.m.f. of discrete distribution.

$$\therefore E(x^2) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x q^{n-x} + \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x} \quad \text{--- (3)}$$

Now solving

$$\sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x q^{n-x} = \sum_{x=2}^{n} \cancel{x(x-1)} \frac{n(n-1)p^2}{\cancel{x(x-1)}} \binom{n-2}{x-2} p^{x-2} \times q^{n-x}.$$

Here we are opening $\binom{n}{x}$ upto two terms

So

$$\sum_{x=2}^{n} n(n-1)p^2 \binom{n-2}{x-2} p^{x-2} q^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x}$$

$$= n(n-1)p^2 \times 1 \quad = n(n-1)p^2 \quad \text{---} (4)$$

as we know that total Probability is always one

$$\sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x} = 1$$

$$\sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{n-x} = 1$$

$$\sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x} = 1$$

and so on.

Similarly

$$\sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x} = \sum_{x=1}^{n} \cancel{x} \frac{n}{\cancel{x}} \binom{n-1}{x-1} p^{x-1} . p . q^{n-x}$$

$$= \sum_{x=1}^{n} np \binom{n-1}{x-1} p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{n-x}$$

$$= np \times 1$$

$$= np \quad \text{---} (5)$$

Putting (4) & (5) in (3)

$$E(x^2) = n(n-1)p^2 + np \quad \text{---} (6)$$

Putting (6) in eqn. (2)

$$\sigma^2 = n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2} = np(1-p)$$

$$V(p) = \cancel{E(p-1)}\,\frac{\sigma^2}{n^2} = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$$

$$S.E.(p) = \sqrt{V(p)} = \sqrt{\frac{p(1-p)}{n}}.$$

* **Sampling distribution of sum of Binomial distribution:-**

Let us suppose that $x_1, x_2, --, x_n$ are indept. Bernoulli random variable that is

$P(X_i=1) = p$ & $P(X_i=0) = 1-P$. Now.

The moment generating function of Bernoulli trials is $M_x(t) = (Pe^t + q) = (q+pe^t)$

So $M_{\Sigma x_i}(t) = \prod_{i=1}^{n} M_{x_i}(t) = (q+pe^t)^n$

which is the m.g.f. of a binomial r.v..

Hence ΣX_i has a binomial distn. with parameter n & P.

* **Sampling distribution of sum Poisson distribution :-**

Let us suppose that $X_1, X_2, --, X_n$ are indept. poisson distributed variable, X_i having parameter λ_i, then

$$M_{X_i}(t) = E[e^{tX_i}] = e^{\lambda_i(e^t-1)}$$

and hence

$$M_{\Sigma x_i}(t) = \prod_{i=1}^{n} M_{x_i}(t) = \prod_{i=1}^{n}\left(e^{\lambda_i(e^t-1)}\right) = e^{\Sigma\lambda_i(e^t-1)}$$

which is again the m.g.f. of a poisson distributed r.v. having parameter $\Sigma\lambda_i$. So the distribution of a sum of indept. poisson distributed r.v.'s is again a poisson distributed r.v. with a parameter equal to the sum of the individual parameters.

Sampling distribution of the mean of a Normal distr. 8

Let $x_1, x_2, --, x_n$ are n independent r.v.'s and

$$X_i \sim N(\mu_i, \sigma_i^2)$$

then $a_i X_i \sim N(a_i \mu_i, a_i^2 \sigma_i^2)$

& $M_{x_i}(t) = e^{\mu_i t + \frac{\sigma_i^2}{2} t^2}$

$$M_{a_i x_i}(t) = e^{a_i \mu_i t + a_i^2 \sigma_i^2 \frac{t^2}{2}}$$

Hence

$$M_{\Sigma a_i x_i}(t) = \prod_{i=1}^{n} M_{a_i x_i}(t)$$

$$= e^{(\Sigma a_i \mu_i) t + \frac{t^2}{2}(\Sigma a_i^2 \sigma_i^2)}$$

which is m.g.f. of a normal variables

so $\Sigma a_i X_i \sim N\left(\sum_{i=1}^{n} a_i \mu_i, \sum_{i=1}^{n} a_i^2 \sigma_i^2\right)$

Thus we can say that "any linear combination i.e. $\Sigma a_i x_i$ of indept. normal r.v.'s is itself a normally distributed random variable.

In particular if X & Y are two r.v.'s

$X \sim N(\mu_x, \sigma_x^2)$ & $Y \sim N(\mu_y, \sigma_y^2)$

then $X+Y \sim N(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$

$X-Y \sim N(\mu_x - \mu_y, \sigma_x^2 + \sigma_y^2)$

iff X & Y are independent.

Similarly if $X_1, X_2, --, X_n$ are indeptly & identically distributed random variable's distributed as $N(\mu, \sigma^2)$ then

$$\overline{X}_n = \frac{1}{n}\Sigma x_i = N\left(\mu, \frac{\sigma^2}{n}\right).$$